Kinematics

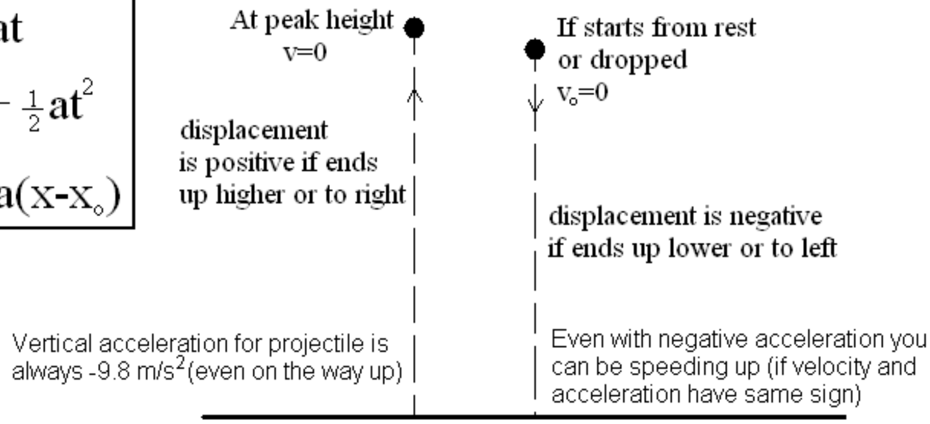
1 D Kinematics

x or y direction

$$v = v_o + at$$

$$x - x_o = v_o t + \tfrac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a(x - x_o)$$



2 D Kinematics

y direction

$$v_y = (v_o \sin\theta) + (-9.8)t$$

$$y - y_o = (v_o \sin\theta)\, t + \tfrac{1}{2}(-9.8)t^2$$

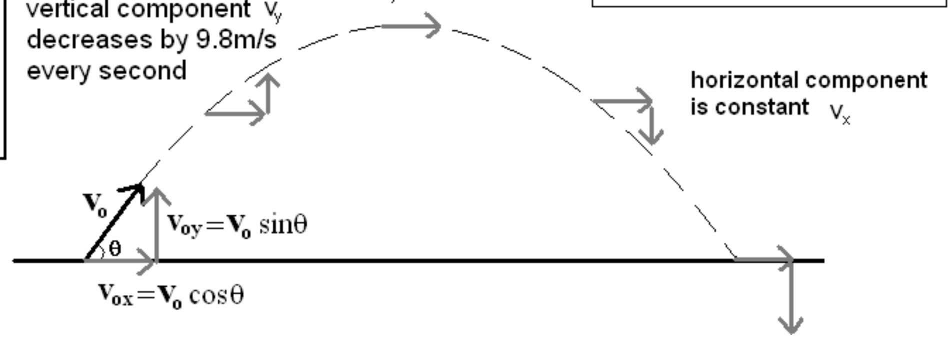
$$v_y^2 = (v_o \sin\theta)^2 + 2(-9.8)(y - y_o)$$

x direction

$$d = (v_o \cos\theta)\, t$$

Range formula (use only if y-yo = 0)

$$R = \frac{v_o^2 \sin(2\theta)}{g}$$

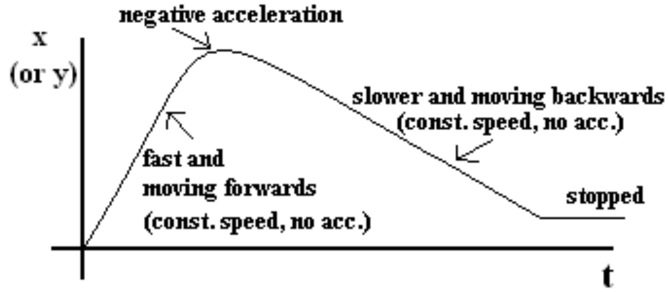


Graphing motion

(Note: the graphs below do not represent the same moving object)

For position vs. time

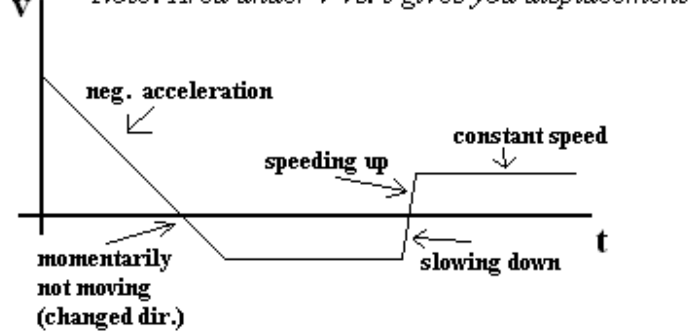
- Slope is velocity
- Steep slope means moving fast
- Curvature is acceleration



For velocity vs. time

- Slope is acceleration
- Going towards v=0 axis means slowing
- Horiz. line means constant speed

Note: Area under v vs. t gives you displacement



Forces

Newton's Laws

1. Objects maintain **constant velocity** unless there is force.

- "No forces" does not mean "no motion". "No forces" means no acceleration (constant motion/velocity).

2. $\Sigma F = ma$

-You need to choose only one direction at a time, e.g. $\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$

3. Every force on an object, has an **equal and opposite** force on a different object.

- Since the forces are on different objects they do not cancel.

Some common kinds of forces

$$F_{gravity} = mg = Gm_1m_2/r^2$$

Note: On another planet $g = GM_p/R^2$

$$F_{normal} = mg \quad \text{unless}$$

1. On incline $F_n = mg\cos\theta$
2. Being pushed down or up $F_n = mg \pm f\sin\theta$
3. In elevator going up or down $F_n = m(g \pm a)$

Note: Scales measure normal force

$$F_{\substack{static\\friction}} \leq \mu_s F_{normal}$$

$$F_{\substack{kinetic\\friction}} = \mu_k F_{normal}$$

$$F_{spring} = kx$$

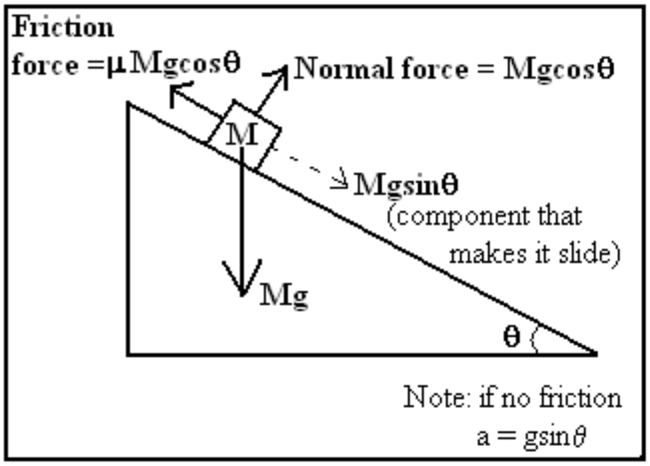
$$F_{buoyancy} = \rho V g$$

$$F_{electric} = kq_1q_2/r^2 = qE$$

$$F_{magnetic} = qvB\sin\theta = ILB\sin\theta$$

Common types of force problems

Inclined plane



Mass pulled by string

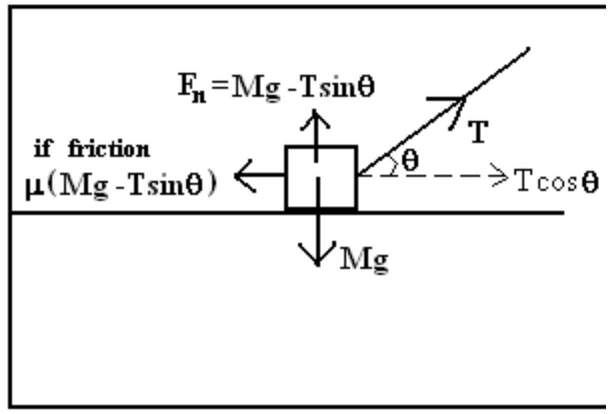
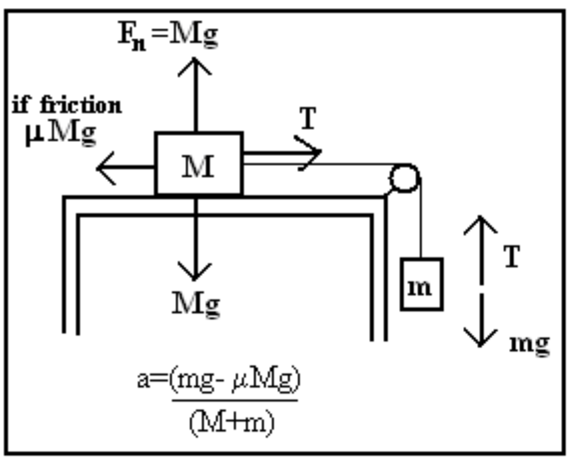
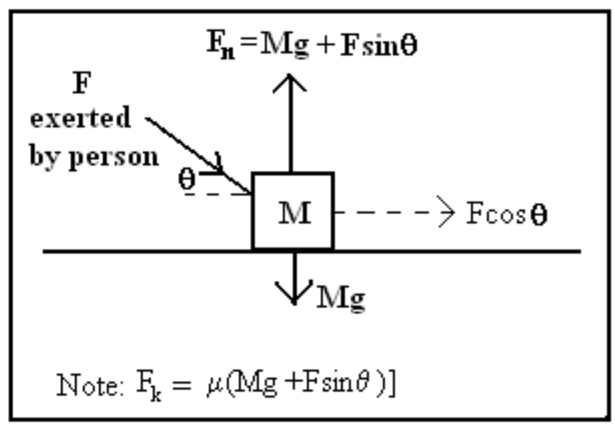


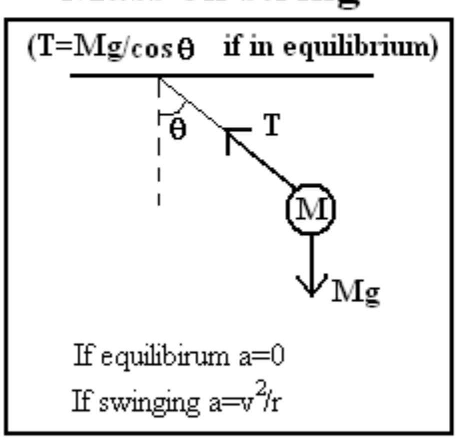
Table and pulley



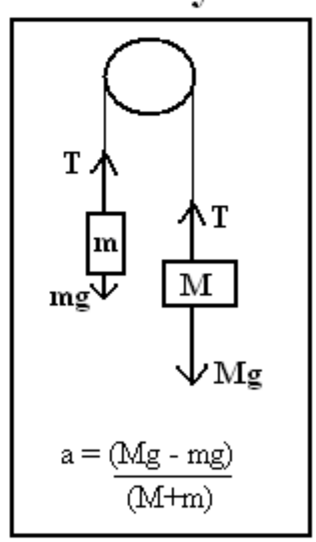
Mass pushed by person



Mass on string



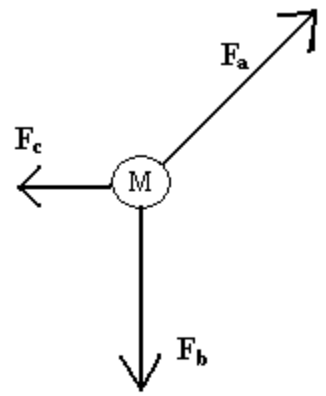
Pulley



Solving 2D force (vector) problems

1. Draw the forces <u>exerted on</u> the object you are concerned with

(These are forces <u>ON</u> the object, not the forces the object is exerting on other objects)

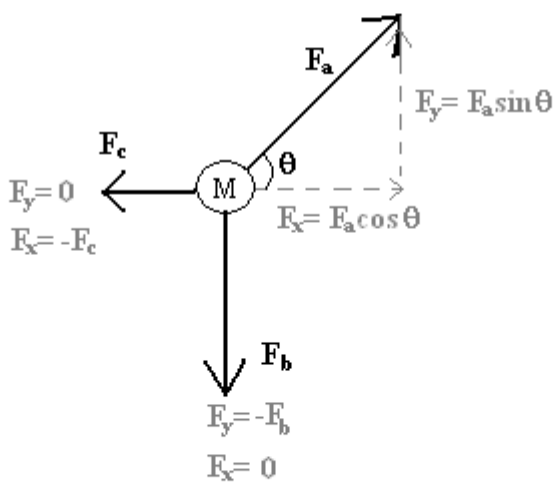


2. Break each force into vertical and horizontal **components F_y and F_x**

(For completely vertical or horizontal forces one component will be zero, the other is $\pm$ F)

(For diagonal forces you need to use sin and cos to break the vector into components)

(Up or Right is a positive component, Left or Down is a negative component)



3. Use $\Sigma F_y = ma_y$ and $\Sigma F_x = ma_x$

i.e. for the example shown $\Sigma F_x = F_a \cos\theta + 0 + (-F_c) = Ma_x$ $\Sigma F_y = F_a \sin\theta + (-F_b) + 0 = Ma_y$

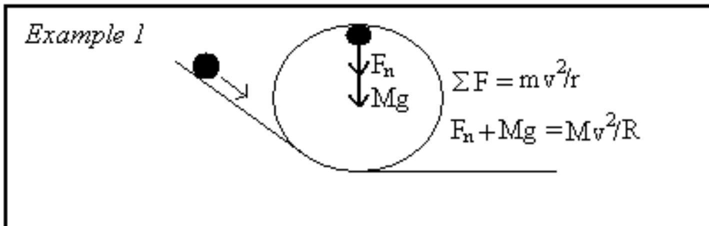
4. If object is not moving in the x direction (or constant speed in x direction) then $\mathbf{\Sigma F_x = 0}$

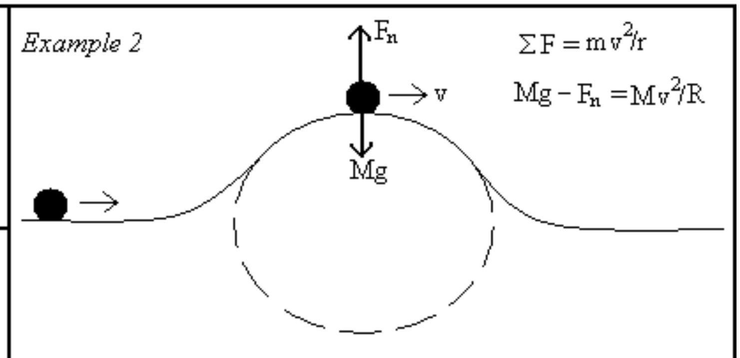
Similarly for the y direction, if the object is not moving vertically (or it is moving at constant speed vertically) then $\mathbf{\Sigma F_y = 0}$

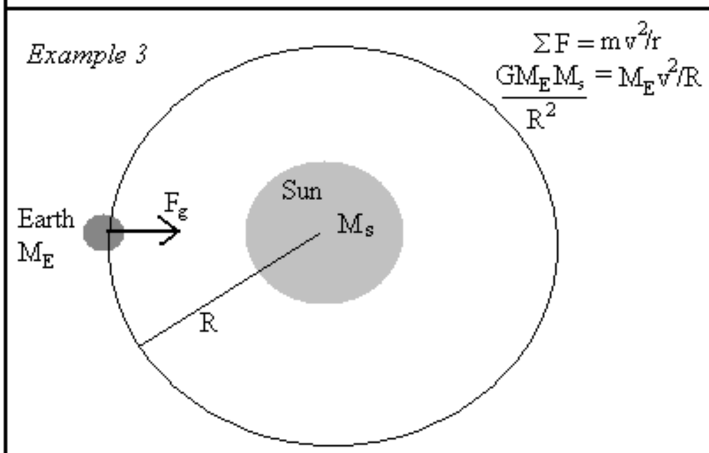
Centripetal Motion

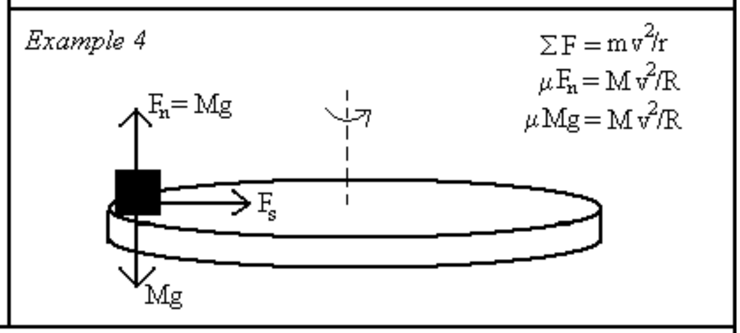
$\Sigma F = m\, v^2/r$

- Forces directed **into the circle** are **positive** and forces directed out of the circle are negative
- You can always use this if something is going in a circle (but you don't always have to use this)
- If an object is moving with constant speed in a circle then **$v = 2\pi r/T_{period}$**



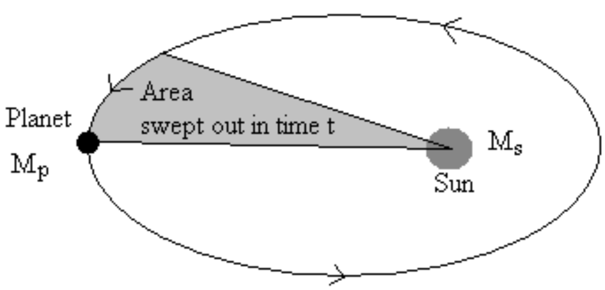






Kepler's Laws

1. Planets follow an ellipse with Sun as one focal point.
2. All planets sweep out equal areas in equal times.
3. The value of R^3/T^2 is the same for each planet.



Note: Kepler's Laws and the 3rd Law derivation work even for objects (moon, satellite) orbiting a planet. You just use mass of planet instead of the mass of the Sun.

Derivation of Kepler's 3rd Law

1. $\Sigma F = m v^2/r$
2. $\frac{G M_p M_s}{R^2} = M_p v^2/R$
3. $\frac{G M_s}{R^2} = v^2/R$ (cancel Mp)
4. $\frac{G M_s}{R^2} = \frac{(2\pi R/T)^2}{R}$ (Sub in formula for v)
5. $\frac{G M_s}{4\pi^2} = \frac{R^3}{T^2}$ (Bring all R's to right side and move $4\pi^2$ to left side)

so $\frac{R^3}{T^2}$ is constant (since Ms, G, and $4\pi^2$ are the same for each planet)