# Kinematics

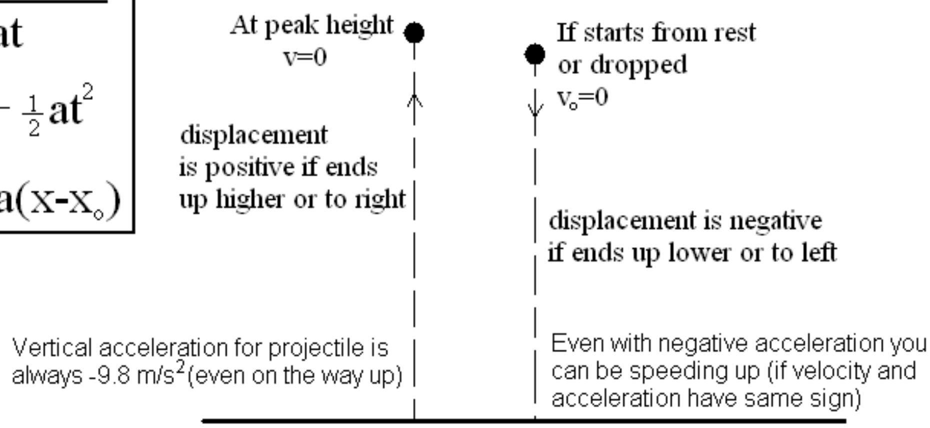
## 1 D Kinematics

*x or y direction*

$$v = v_o + at$$

$$x - x_o = v_o t + \tfrac{1}{2} a t^2$$

$$v^2 = v_o^2 + 2a(x - x_o)$$




## 2 D Kinematics

*y direction*

$$v_y = (v_o \sin\theta) + (-9.8)t$$

$$y - y_o = (v_o \sin\theta)\, t + \tfrac{1}{2}(-9.8) t^2$$

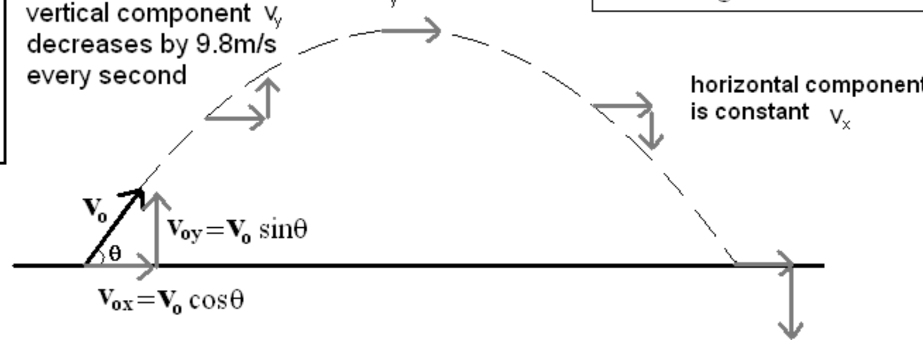
$$v_y^2 = (v_o \sin\theta)^2 + 2(-9.8)(y - y_o)$$

*x direction*

$$d = (v_o \cos\theta)\, t$$

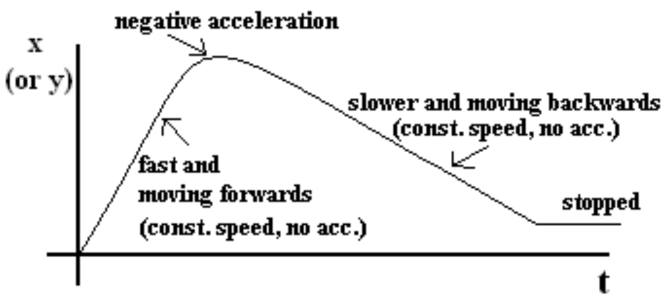
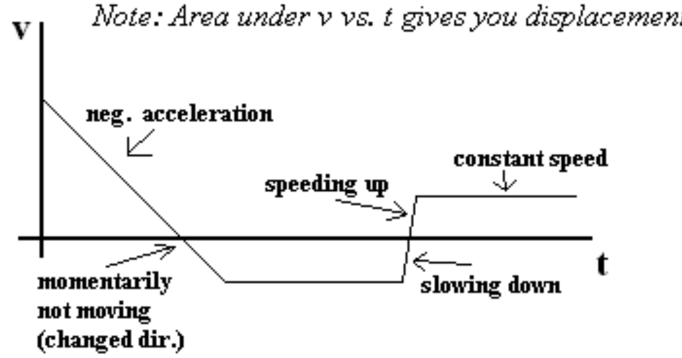
Range formula (use only if $y - y_o = 0$)

$$R = \frac{v_o^2 \sin(2\theta)}{g}$$




## Graphing motion

(Note: the graphs below do not represent the same moving object)

| For position vs. time | For velocity vs. time |
| --- | --- |
| - Slope is velocity<br>- Steep slope means moving fast<br>- Curvature is acceleration | - Slope is acceleration<br>- Going towards v=0 axis means slowing<br>- Horiz. line means constant speed<br>*Note: Area under v vs. t gives you displacement* |
|  |  |

# *Forces*

## Newton's Laws

### **1.** Objects maintain **constant velocity** unless there is force.

- "No forces" does not mean "no motion". "No forces" means no acceleration (constant motion/velocity).

### **2.** $\Sigma F = ma$

-You need to choose only one direction at a time, e.g. $\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$

### **3.** Every force on an object, has an **equal and opposite** force on a different object.

- Since the forces are on different objects they do not cancel.

## Some common kinds of forces

$$F_{gravity} = mg = Gm_1m_2/r^2$$

Note: On another planet $g = GM_p/R^2$

$$F_{normal} = mg \quad \text{unless}$$

1. On incline $F_n = mg\cos\theta$
2. Being pushed down or up $F_n = mg \pm f\sin\theta$
3. In elevator going up or down $F_n = m(g \pm a)$

*Note: Scales measure normal force*

$$F_{\substack{static\\friction}} \leq \mu_s F_{normal}$$

$$F_{\substack{kinetic\\friction}} = \mu_k F_{normal}$$

$$F_{spring} = kx$$

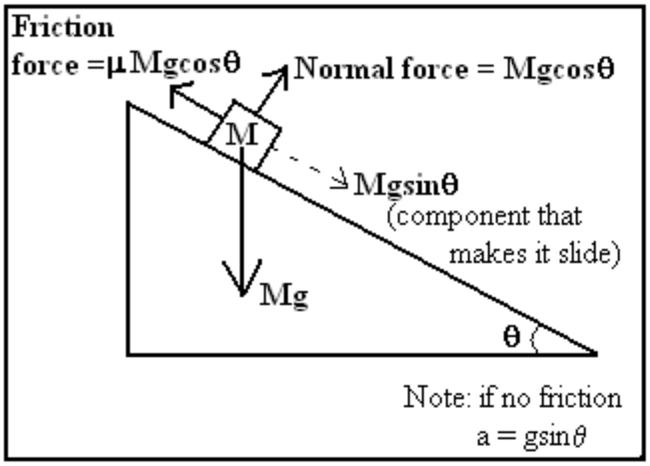
$$F_{buoyancy} = \rho V g$$

$$F_{electric} = kq_1q_2/r^2 = qE$$

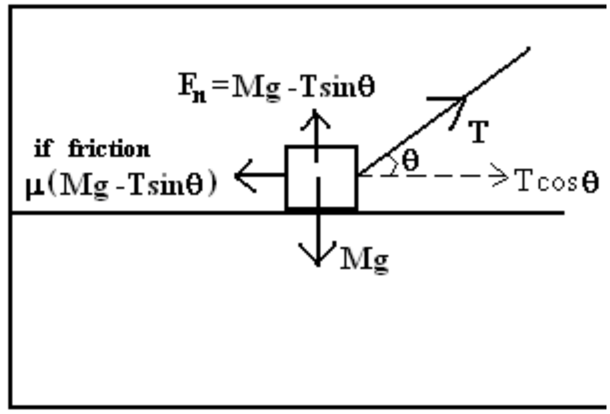
$$F_{magnetic} = qvB\sin\theta = ILB\sin\theta$$

# Common types of force problems

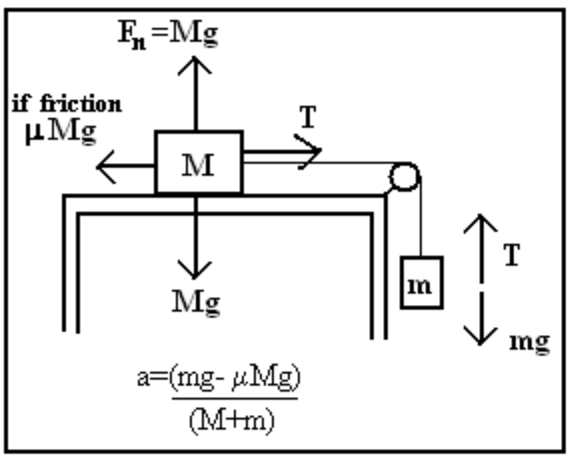
## Inclined plane



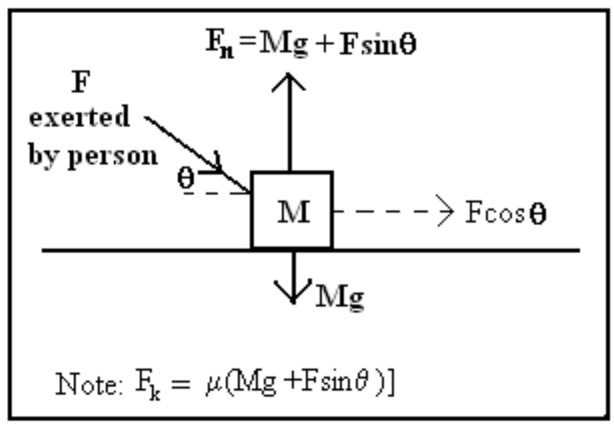

## Mass pulled by string



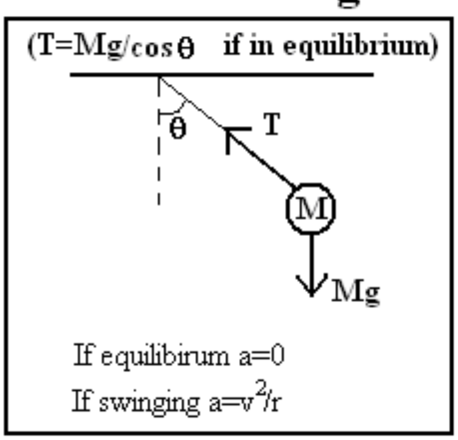

## Table and pulley



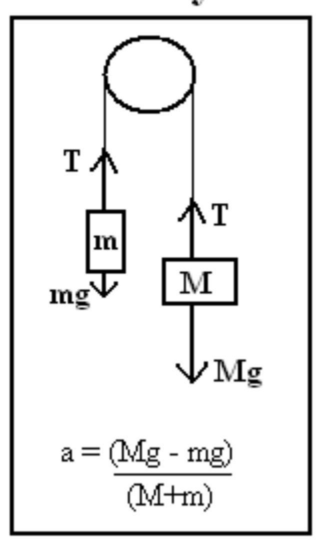

## Mass pushed by person




## Mass on string



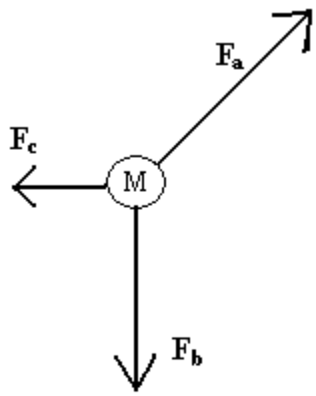

## Pulley



# Solving 2D force (vector) problems

**1. Draw the forces** <u>exerted on</u> the object you are concerned with

(These are forces <u>ON</u> the object, not the forces the object is exerting on other objects)

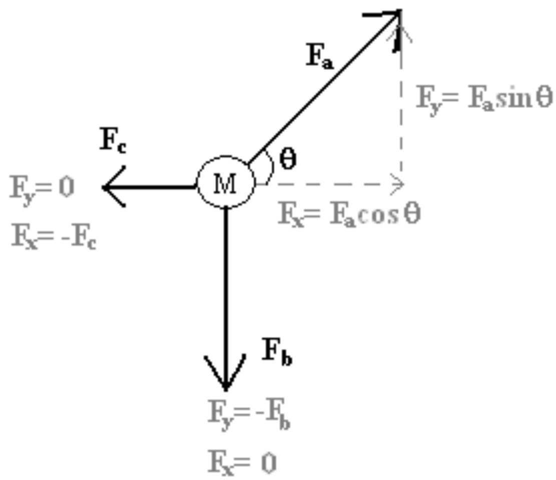



**2.** Break each force into vertical and horizontal **components $F_y$ and $F_x$**

(For completely vertical or horizontal forces one component will be zero, the other is $\pm$ F)

(For diagonal forces you need to use sin and cos to break the vector into components)

(Up or Right is a positive component, Left or Down is a negative component)




**3.** Use $\boldsymbol{\Sigma F_y = ma_y}$ and $\boldsymbol{\Sigma F_x = ma_x}$

i.e. for the example shown $\Sigma F_x = F_a \cos\theta + 0 + (-F_c) = Ma_x$ $\Sigma F_y = F_a \sin\theta + (-F_b) + 0 = Ma_y$

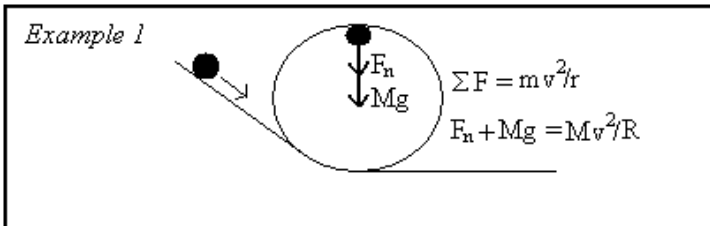
**4.** If object is not moving in the x direction (or constant speed in x direction) then $\boldsymbol{\Sigma F_x = 0}$

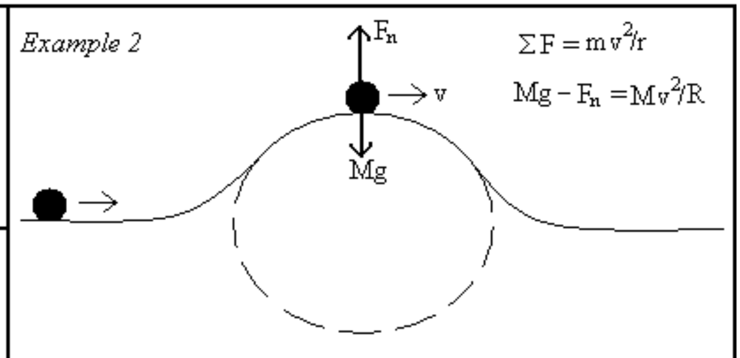
Similarly for the y direction, if the object is not moving vertically (or it is moving at constant speed vertically) then $\boldsymbol{\Sigma F_y = 0}$

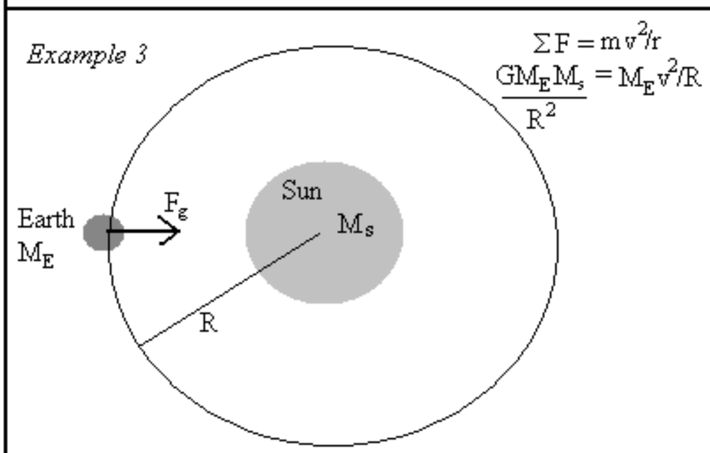
# Centripetal Motion

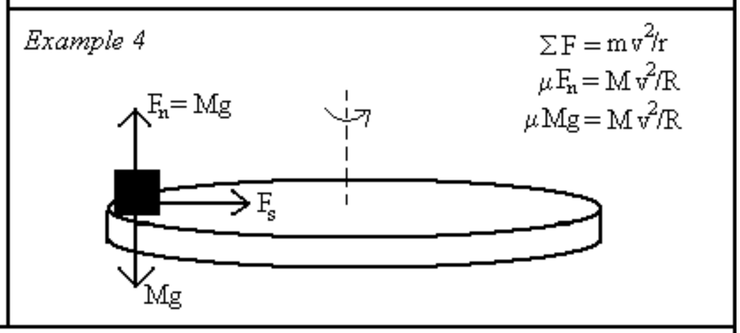
## $\Sigma F = m v^2/r$

- Forces directed **into the circle** are **positive** and forces directed out of the circle are negative
- You can always use this if something is going in a circle (but you don't always have to use this)
- If an object is moving with constant speed in a circle then $v = 2\pi r/T_{period}$



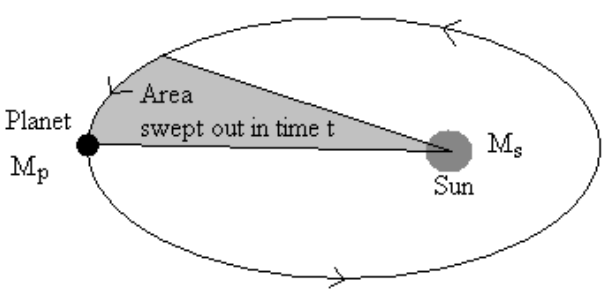




Kepler's Laws

1. Planets follow an ellipse with Sun as one focal point.
2. All planets sweep out equal areas in equal times.
3. The value of $R^3/T^2$ is the same for each planet.




Derivation of Kepler's 3rd Law

1. $\Sigma F = m v^2/r$
2. $\frac{G M_p M_s}{R^2} = M_p v^2/R$
3. $\frac{G M_s}{R^2} = v^2/R$ (cancel Mp)
4. $\frac{G M_s}{R^2} = \frac{(2\pi R/T)^2}{R}$ (Sub in formula for v)
5. $\frac{G M_s}{4\pi^2} = \frac{R^3}{T^2}$ (Bring all R's to right side and move $4\pi^2$ to left side)

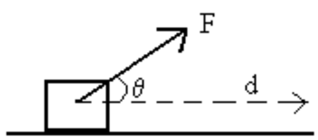
so $\frac{R^3}{T^2}$ is constant (since Ms, G, and $4\pi^2$ are the same for each planet)

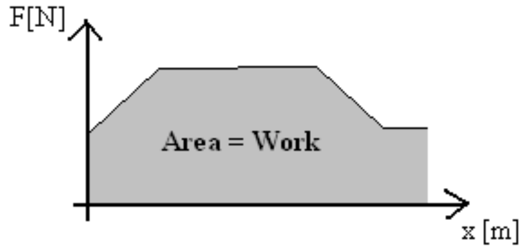
Note: Kepler's Laws and the 3rd Law derivation work even for objects (moon, satellite) orbiting a planet. You just use mass of planet instead of the mass of the Sun.

# *Energy and Work*

## Work

**$W=Fd\cos\theta_{Fd}$** (***Note: Area under F vs. x graph equals work***.)




- Work tells you how much energy is tranfered
- If a force pushes in the direction of d (tries to speed up object), force does + work
- If the object doesn't move d=0, or the force is perpendicular to motion, **W=0**
- The work done by gravity is **$W_g = \pm mgh$** (use + if moving down, - if moving up)
- Work-Kinetic energy **$W_{total} = \Delta KE = KE_f - KE_i$**

## Types of energy

$KE = \frac{1}{2}mv^2$

$PE_{gravity} = mgh$ (you choose where h=0)

$PE_{spring} = \frac{1}{2}kx^2$ (x is the compression or extension)

**Thermal Energy =**
use W= **Fd**

- $\mu F_n d$ for friction
- $F_{air} d$ for air resistance
- or just determine how much mechanical energy was lost $(PE+KE)_i - (PE+KE)_f$

$Q_{heat} = mc\Delta T = mL$

$PE_{electric} = qV = kq_1q_2/r$

$E_{capacitor} = \frac{1}{2} CV^2 = \frac{1}{2} QV$

$E_{photon} = hf$

$E = mc^2$

# *Momentum and Collisions*

## Momentum

## $p = mv$    [kg m/s]

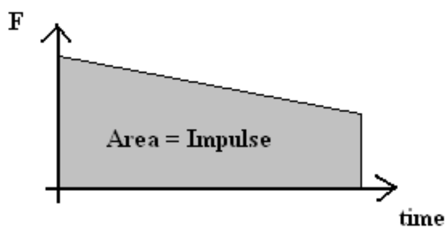
-Momentum is a vector and direction matters (+ if going right, - if going left)
-A situation where two objects of mass m move with speed v toward each other has no total momentum
-Momentum is always conserved

### $\Sigma F = \Delta p/\Delta t$    [which also = ma]

-This is a way to relate changes in momentum to forces

### Impulse = $\Delta p = F\Delta t$    [Ns]

-Area under **F vs. time** graph is the Impulse (change in momentum)




## Collisions

-momentum is conserved in every type of collision (elastic and inelastic)

### a. Elastic collisions

- KE is conserved
-objects must bounce off each other
**-Note: For elastic collisions, $(v_1 - v_2)_{initial} = -(v_1 - v_2)_{final}$**

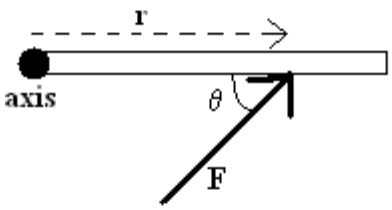
### b. Inelastic collisions

- KE is not conserved, KE gets lost in the collision (turns into thermal energy, etc)
-objects can stick together (***perfectly inelastic***) or bounce off each other
**Note: To determine whether a collision is elastic or inelastic just compare the $KE_{tot}$ before and after the collision.**
- If $KE_{tot\ initial} = KE_{tot\ final}$, then it is elastic. If not, it is inelastic.

# *Torque and Center of Mass*

## Torque

$$\tau = rF\sin\theta_{rF}$$




- r is from the axis to the point where the force is applied

## Equilibrium

$$\Sigma F = 0 \qquad \Sigma\tau = 0$$

- no acceleration and no angular acceleration (usually means at rest)
- i.e. Torque in CW direction equals torque in CCW direction

## Center of mass

$$CM = \frac{1}{M_{tot}}\Sigma mr = \frac{1}{M_{tot}}\left(m_1r_1 + m_2r_2 + \ldots\right)$$

- r is distance from arbitrary point (end of object, middle of object, etc.) to mass m or dm, but if there is an obvious axis it is usually a good idea to use it

- the CM tells you the position where an object could balance

- the CM is also where you could treat all the mass as residing

# *Waves and Simple Harmonic Motion*

## Simple Harmonic oscillators

- position described by **$x(t)=A\sin(\omega t)$** or **$x(t)=A\cos(\omega t)$** $\omega = 2\pi/T$ [rad/sec]

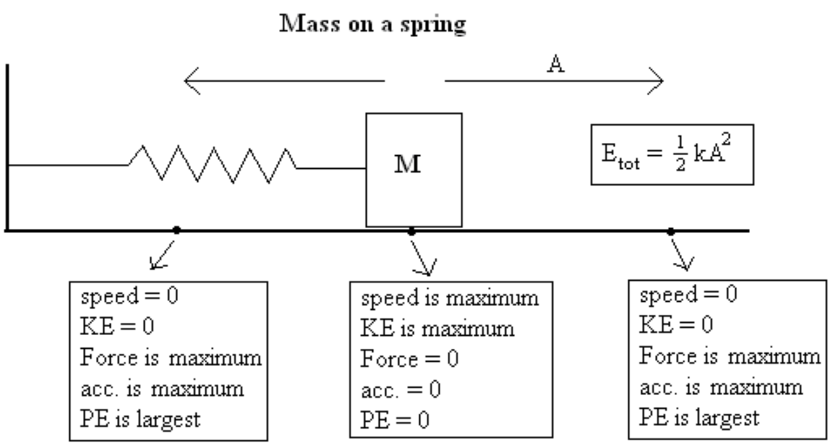
**$T_{mass\ on\ spring} = 2\pi\ [m/k]^{1/2}$** Remember that **$f = 1/T$** [Hz]

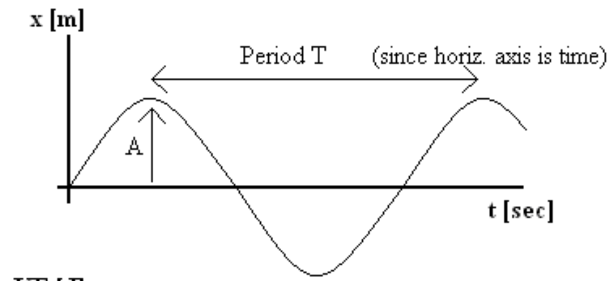
-Spring Period T does not depend on g or amplitude

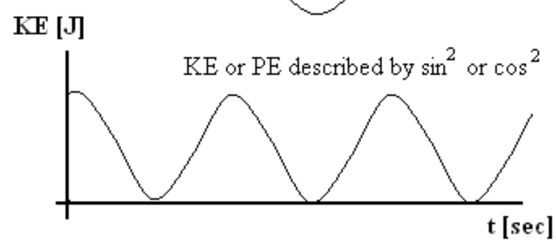
- $E_{total} = \frac{1}{2} kA^2$ ( for horiz. mass on spring, because at the maximum displacement A the KE=0 )

**$T_{pendulum} = 2\pi\ [L/g]^{1/2}$**

- Pendulum Period T does not depend on mass or amplitude (as long as the amplitude is small $\theta < 20^o$)




## Waves

**$v = \lambda f$** (v is wave speed, λ is wavelength, f is frequency) **Note: this eqn. is true for every wave**

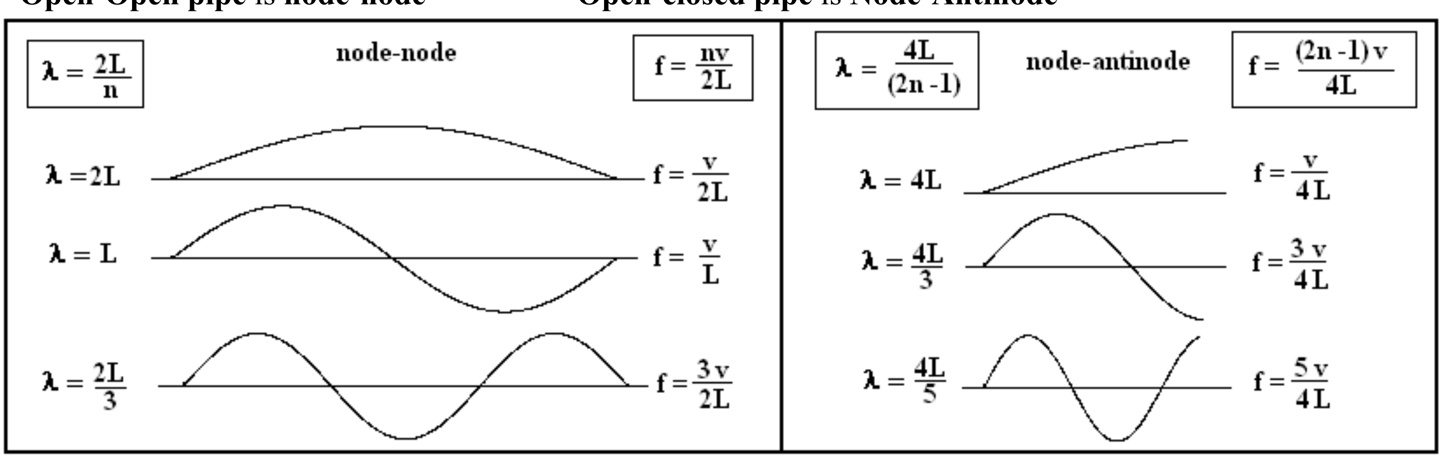
**Transverse waves:** medium moves perpendicular to velocity of wave (e.g. ripple on pond)

**Longitudinal waves:** medium moves parallel to velocity of wave (e.g. sound waves)

### Standing waves

- **Nodes**: points that don't move (dest. int) **Antinodes**: points of maximum displacement (const. int)

- **Open-Open pipe** is **node-node** **Open-closed pipe** is **Node-Antinode**



# *Circuits*

## Current

## $I = Q/t$ [C/sec = Amperes]

- defined to be in the direction of positive charge flow (or opposite direction of $e^-$)
- is directed out of the + terminal of a battery, and into the - terminal

## Resistance

The resistance of a length **L** of cylinder made with resistivity **ρ**, and cross sectional area **A** is,

## $R = \rho L/A$ [Ohms]

## Ohm's Law

## $V = IR$ (V is voltage drop across resistor, I is current through the resistor, R is resistance)

- V is not necessarily the voltage of the battery!
- **Ohmic materials** have constant resistance (slope on V vs. I), regardless of what the current is
- **Non-Ohmic materials** change their "resistance" depending on what the current/voltage is

## Electrical Power

## $P = IV$ [Watts]

## $P = I^2R$

## $P = V^2/R$

## Capacitors

## $C = Q/V$ (C is capacitance, Q is charge on + plate, V is voltage across capacitor)

- Capacitance tells you how well a capacitor can store charge
- Inserting a **Dielectric** between a capacitor always **increases capacitance** by a factor of **k**
- Capacitors store energy as well, which is given by

## $E_{capacitor} = ½ QV = ½ CV^2$

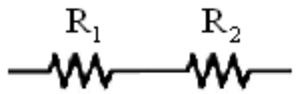
- For a parallel plate capacitor with plates of area A separated by a distance d, capacitance is,

## $C = \varepsilon_o A/d$

## Combining Resistors

### Resistors in Series

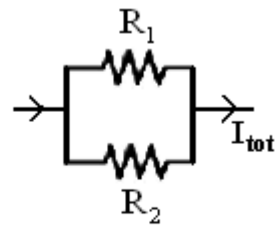
$$R_{eq} = R_1 + R_2$$




Note: Resistors in Series always have same current

### Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$




Note: Resistors in Parallel always have same voltage

**If circuit has only 1 battery,**

Choose resistors, two at a time, and reduce to a single resistor to determine the current through the battery. Then determine how current breaks up at junctions using these rules ↗

If one resistor is 3 times larger than the other, smaller resistor gets 3/4 of the total current $I_{R_1} = \frac{3}{4} I_{tot}$

If one resistor is 5 times larger than the other, smaller resistor gets 5/6 the total current $I_{R_1} = \frac{5}{6} I_{tot}$

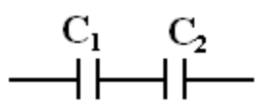
or, if resistors are not a nice ratio use this formula

$$I_{R_1} = I_{tot} \frac{R_2}{(R_1 + R_2)}$$

## Combining Capacitors

### Capacitors in Series

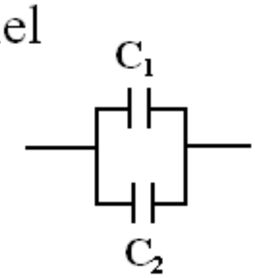
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$




Note: Capacitors in series all have same charge

### Capacitors in Parallel

$$C_{eq} = C_1 + C_2$$




Note: Capacitors in parallel all have same voltage

When you reduce all capacitors to a single C, you can find Q (=CV). Then work backwards to find Q on each capacitor.

For capacitors in parallel, if one capacitor has **3** times more capacitance than the other, it gets **3/4** of the total charge.

Or, if not a nice ratio use,

$$Q_1 = Q_{tot} \frac{C_1}{C_1 + C_2}$$

Note: After a short time, current will no longer flow through a C, and any segment of a circuit with a C will have no current.

# Kirchoff's Rules

## Junction Rule: $I_{in} = I_{out}$

- Total current flowing into junction equals total current flowing out of junction

## Loop Rule: $\Sigma\Delta V = 0$

- The sum of the changes in voltage around any closed loop always equals zero

$\Delta V = -IR$ (if you **pass through** resistor in the **same direction as current**)

$\Delta V = IR$ (if you **pass through** resistor in the **opp. direction as current**)

$\Delta V = +\varepsilon_{battery}$ (if you **pass through** the battery from – **terminal to** + **terminal**)

$\Delta V = -\varepsilon_{battery}$ (if you **pass through** the battery from + **terminal to - terminal**)

# Terminal Voltage

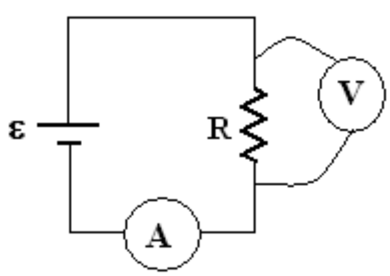
## $V_{ab} = \varepsilon - Ir$ ($V_{ab}$ is the terminal voltage, $\varepsilon$ is the emf of battery, **r** is internal resistance)

-Every battery has an internal resistance **r** which will lower the terminal voltage when current flows
- A 9V battery will not necessarily have a measured terminal voltage of 9V, unless no current flows
- The $\varepsilon$ of a 9V battery is 9V even when no current flows, but the measured terminal voltage will be less
- **Slope** of Vab vs. I graph is negative the **internal resistance**. The **y intercept** is the **emf** $\varepsilon$.

# Electrical Meters

## Voltmeter

- Measures voltage change across circuit element (resistor, battery, etc.)
- Ideally has **infinite resistance** so it does not draw any current away from circuit
- Needs to be hooked up in **parallel** with circuit element




## Ammeter

- Measures current through a circuit element (resistor, battery, etc.)
- Ideally has **no resistance** so it does not change the current
- Needs to be hooked up in **series** with circuit element